A Note on the Quantum Creation of Universes

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ABSTRACT: We elucidate the nature of the correction to the Hartle-Hawking wavefunction presented in hep-th/0505104 and hep-th/0406107. The correction comes from the quantum fluctuation of the metric that spontaneously breaks the classical deSitter symmetry. This converts the tunneling from nothing to a deSitter-like universe via a S^4 instanton to that via a barrel instanton, which is bounded from below. Its generalization to 10 dimensional spacetime allows us to find the preferred sites in the stringy cosmic landscape. We comment on how some of the problems of the Hartle-Hawking wavefunction are avoided with the new modified wavefunction of the universe, when applied to the spontaneous creation of an inflationary universe. We also summarize our arguments on the validity of the Hartle-Hawking wavefunction in the minisuperspace approximation, as opposed to the WKB formula suggested by Linde and Vilenkin.

Contents

1. Introduction and Summary

In recent years, it has become clear that the vacuum structure in superstring theory is very rich. The number of metastable discrete vacua is estimated to be somewhere between 10^{300} to ∞. Different vacau have different forces and matter content. This is the cosmic landscape. With these many possible vacuum states, one is led to ask why we end up where we are, in a particular vacuum. One may choose to believe in the (strong) anthropic principle and so avoid addressing this very deep and fundamental question. Alternatively, one may try to find a physics reasoning to gain a better understanding of the questions "why and how".

In an attempt to address this question, we start with Vilenkin's beautiful idea of tunneling from nothing [1], or equivalently, the no-boundary wave function of Hartle and Hawking [2]. Here, nothing means no classical space-time. Generalizing to 10-dimensional spacetime, one can calculate the tunneling probability from nothing to every state in the cosmic landscape in superstring theory. Our universe should begin
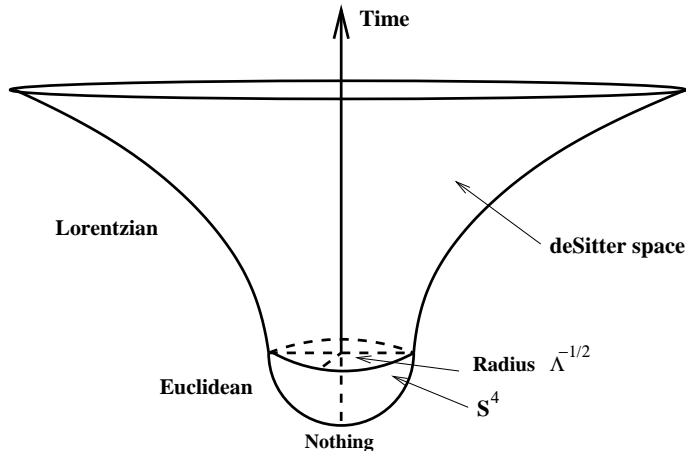
Figure 1: The creation of a closed deSitter universe from nothing.

with the state that has the maximum (or close to the maximum) tunneling probability from nothing. Hopefully, one finds that our inflationary universe is among the ones preferred. If that inflationary universe then evolves to our today's universe, one would have a physical understanding why we end up where we are. As a byproduct, we should also understand why our observable universe has a 3+1 dimensional observable space-time. Unfortunately, the Hartle-Hawking (HH) wavefunction of the universe does not seem to give the answer we are looking for.

Our proposal [3, 4] introduces a correction term to the HH wavefucntion for a closed universe. Since tunneling is a quantum process, consistency requires the inclusion of quantum corrections to the resulting universe as well. The correction term is simply the result of including this quantum correction. By going a little outside the mini-superspace approximation, this correction remedies the problem faced by the HH wavefunction. We can also see how this correction emerges in an explicit calculation. Although the improved wavefunction and its application to the landscape concerns 10-dimensional spacetime in string theory, we shall simplify the discussion here by focusing on 4-dimensional Einstein theory.

In the usual evaluation of the quantum one-loop correction to the global Euclidean deSitter space, one constrains the quantum correction to obey the symmetry of the S^4 instanton (i.e., the deSitter $SO(5)$ rotational symmetry). As a consequence of this choice of renormalization condition, the only possible correction is a renormalization of the cosmological constant (and/or of the curvature term) in pure de-Sitter space. However, there is no apriori reason for this. In fact, the deSitter space has a horizon and so an effective Hawking temperature. So, quantum fluctuations naturally introduce tensor modes as metric perturbations (a well-known fact in the inflationary universe). Here, such modes play the role of radiation, and their presence brings us outside the mini-superspace approximation. Taking into account

its back reaction on the metric, this perturbation breaks the pure deSitter symmetry. Although mathematically one can get a pure quantum deSitter space, it is more natural physically to choose the renormailzation condition so that the classical pure deSitter symmetry is broken by the quantum effect. Our key position is that there is no pure quantum deSitter space in this physical sense. As a result, there is no tunneling to a pure deSitter universe. Our calculation shows that this spontaneous breaking of the deSitter symmetry converts the S^4 instanton to a barrel instanton [4].

The quantum fluctuation appears as radiation in Lorentzian metric. Integrating out the metric perturbative modes in Euclidean metric, we find a new term that corrects the HH wavefunction. Instead of the S^4 instanton, we now end up with a barrel instanton. In terms of the tunneling probability from nothing to a quantum-corrected deSitter universe with cosmological constant Λ, the radiation introduces a new term to the Euclidean action,

$$P \sim \exp(-S_E) = \exp(3\pi/G\Lambda) \rightarrow \exp\left(\frac{3\pi}{G\Lambda} - \frac{bM_s^4}{\Lambda^2}\right) \qquad (1.1)$$

where $G = M_{Planck}^{-2}$ is the Newton's constant, b is a numerical constant and M_s is the ultra-violet cut-off. This cut-off breaks the conformal property. In string theory, this UV cut-off is provided by the string scale. Since we are considering a closed universe, there is no modes with wavelength longer than the size of the closed universe. So there is a natural infrared cut-off in the wave number of the metric perturbative modes, namely, the finite size $\sim \Lambda^{-1/2}$. This leads to the Λ^{-2} factor. Besides the tensor modes, all fields with mass smaller than M_s will contribute, so b depends on the specific matters fields present. Maximizing the tunneling probability, one finds $\Lambda_{max} = 2bM_s^4/3\pi M_{Planck}^2$. Typically the string scale is below the Planck scale, and the logarithm of b is of order unity, so Λ_{max} is much smaller than the Planck scale. In string theory, different sites in the landscape with 3 large spatial dimensions will have different values of b. Recall that, without the new term (i.e., the original HH wavefunction), $\Lambda = 0$ would have been preferred. This is due to the unboundedness of the Euclidean action S_E. On the other hand, the improved barrel instanton action is bounded from below.

In Ref [3], we use the concept of decoherence to argue for the existence of such a correction. In Ref [4], the calculation of the barrel instanton is motivated along this line of thinking. The metric perturbation that leads to the radiation term occurs over a Hubble time and so is effectively non-local. This radiation acts as the environment to the cosmic scale factor a (the system) and thus suppresses the tunneling, i.e., the spontaneous creation of the universe from nothing. Based on the well-understood idea of decoherence in quantum measurement [5], this is precisely what one expects. As Λ decreases, there are more modes contributing to the environment, hence further suppression. Also, the presence of more matter fields tends to increase the suppression of the tunneling probability as well, so b measures the

degrees of freedom. Extending the wavefunction to 10-dimensions is straightforward. In the case where 6 dimensions are compactified, the compactification properties are encoded in G, Λ and b in Eq(1.1).

One can now use this improved wavefunction (i.e., its generalization to 10-dimensions) to study the spontaneous creation of universes and find out why some vacua are preferred over others. Among known string vacua (such as any super-symmetric vacuum, a 10-dimensional deSitter-like vacuum etc.), a 3+1 dimensional inflationary universe not unlike ours seems to be preferred. Here we have in mind, for example, the brane inflationary scenario in the brane world in string theory [3,6,7]. It is important to note that, instead of our universe today (with a tiny cosmological constant), the tunneling probability picks an inflationary universe, which can then evolve to our universe today. We refer to this as SOUP, the selection of the original universe principle. This preliminary result is attractive enough to deserve further analysis. In fact, the barrel instanton seems to have distinct cosmological signatures that, at least in principle, may be detected in the cosmic microwave background radiation [8].

It is obviously important to understand the precise meaning and the properties of what we refer to as "nothing", with no classical spacetime. Since spacetime appears to be a derived quantity in string theory, we expect "nothing" to be fully meaningful in string theory. Just like quantum field theory provided us a good understanding of the vacuum in the past century, string theory should provide us a good understanding of the full quantum nature of "nothing". Conceptually, this may be the ultimate challenge in string theory. Here, the tunneling probability is calculated in the semi-classical approximation. Fortunately, the vacua with high probabilities are precisely the ones with the effective cosmological constant small compared to the Planck scale, so the semi-classical approximation should be valid here. So we are hopeful that the improved wavefunction in its semi-classical approximation will allow us to select the preferred site in the cosmic landscape.

As mentioned earlier, the original HH wavefunction seems to prefer a zero cosmological constant Λ, corresponding to an infinite size universe ($a \sim \Lambda^{-1/2}$, thus skipping the big bang era). This clearly disagrees with observation, is counter-intuitive, and, some believe [9,10], inconsistent. This problem motivated Linde and Vilenkin to seek an alternative tunneling formula or wavefunction [11], which we call the LV wavefunction. Since our new wavefunction is predicated on the HH wavefunction, not on the LV wavefunction, it is important for us to explain the correctness of the HH wavefunction as the zeroth order approximation. This is one of the purposes of this note.

In quantum field theory, the tunneling probability from one state to another state is well-understood. One may use the instanton method, where the tunneling probability P is given by the Euclidean action S_E, namely $P \simeq e^{-S_E}$ as in HH wavefunction. Alternatively, one may use the WKB method in the Wheeler-DeWitt

formalism, where the tunneling amplitude is given in turns of the integral of the momentum across the barrier. It is understood that typical tunneling process is exponentially suppressed, so the negative sign is chosen, as in the LV wavefunction, so $P \simeq e^{-2 \int |p| dr}$. In quantum field theory, these 2 approaches yield the same answer. However, when applied to gravity, the Euclidean action can be unbounded from below. In fact, for tunneling from nothing to a closed deSitter universe, $S_E = -3\pi/G\Lambda$ where Λ is the cosmological constant and G is the Newton constant. In the instanton method, this naively seems to lead to an exponentially large tunneling probability for the spontaneous creation of a universe as $\Lambda \to 0$, while the LV wavefunction strongly disfavors this limit. This leads to a puzzle, as illustrated in Fig. 2. This is the 20 years old debate on which sign is correct [12].

Our proposal introduces a new term to the HH wavefunction, implying that the HH wavefunction is the correct starting point, not the alternative LV version. Furthermore, the improved wavefunction no longer suffers from the above-mentioned problems that originally led to the consideration of alternatives. Looking at the WKB approximation, we see that the other WKB solution, namely, $P \simeq e^{+2 \int |p| dr}$ is the correct choice for the tunneling processes here.

In this note we like to further clarify the modified wavefunction and collect all the arguments, some old and some new, for the HH version and against the LV version :

• In the instanton formulation, Lorentzian time is rotated to Euclidean time via $t \to -i\tau$. For quantum fields, this leads to a consistent formulation, where a realistic scalar field Euclidean action will be bounded from below. In the LV version, one has to rotate time the other way, i.e., $t \to i\tau$. In this case, the scalar field Euclidean action will be unbounded from below. In pure gravity, such an unusual Wick rotation may be acceptable. However, any realistic stringy vacuum or gravity theory will include scalar fields (say, the moduli in string vacua and the inflaton field in an inflationary model). So consistency of the scalar field will dictate the correct Wick rotation, which chooses the HH version. The alternative will cause catastrophic behavior in the scalar fields. This issue has been discussed by Rubakov and others [13].

• Recently, a topological string theory calculation yields the HH wavefunction, not the LV wavefunction. In Ref [14], Ooguri, Vafa and Verlinde argue that the topological string partition function can be interpreted as an exact HH wavefunction of the universe in the minisuperspace sector of the physical superstring theory, including all loop quantum corrections. Although they derived the wavefunction of the universe in a cosmology with a negative cosmological constant, it can be seen as the derivation of a Hartle-Hawking like result in string theory. The geometry considered is a Euclideanized version of $AdS^2 \times S^2 \times M$, where M is a Calabi-Yau 6-fold; the result includes all loop corrections. Their proposal leads to a conceptual understanding of the observation that the black hole entropy is the square of the topological string wavefunction. The wavefunction thus obtained prefers a universe with vanish-

	WKB	**INSTANTON METHOD**
QFT	$e^{-2\int\sqrt{2V(\phi)}\,d\phi}$	$= \quad \int D[\phi]\,e^{-S_E[\phi]}$
	$\big\downarrow$	$\big\downarrow$
GRAVITY	$e^{-2\int\sqrt{2V(a)}\,da}$	$\neq \quad \int D[a]\,e^{-S_E[a]}$

Figure 2: Debate about the correct sign in the exponent in the tunneling probability from nothing to a closed deSitter (inflationary) universe. The WKB and the instanton approaches agree in quantum field theories. However, due to the negative sign of $S_E[a]$ (and its unboundedness from below), they seem to disagree in the quantum creation of the universe. We argue that the instanton approach, which in the zero-order approximation corresponds to the Hartle-Hawking wavefunction, is correct. That is, one should take the other (i.e., the plus sign term) WKB solution in this particular application.

ing cosmological constant. Although their result is for topological string theory in the minisuperspace approximation, the preference for HH over LV wavefunction will most likely survive in the physical superstring theory.

• In Ref. [4], we use decoherence to find the correction term to the HH wavefunction. Going beyond the mini-superspace, we introduce fluctuations around the background metric as the environment where the scale factor a is the system. This may be considered as a quantum "back-reaction improved" 1-loop effect. The tunneling is then suppressed by its interaction with the environment. This is expected. The tunneling is from nothing to a universe with size $a \simeq H^{-1} \simeq 1/\sqrt{\Lambda}$. As $\Lambda \to 0$, the resulting universe becomes infinite. The tunneling to such a super-macroscopic universe must be suppressed. Indeed, this suppression is the decoherence effect.

Note that the quantum fluctuation considered here has the same origin as the tensor perturbation studied in an inflationary universe. There, one is interested in the tensor mode with wave number $k = aH$ as it leaves the horizon. Here we are interested in all modes. It is the sum over all modes that requires the introduction of an ultra-violet cut-off. After tunneling, their contribution behaves as a radiation term, which fixes the sign of this correction to the S^4 instanton. If we apply the

decoherence effect to the LV wavefunction, this environment actually enhances the tunneling, contrary to our expectation from decoherence. Hence the decoherence interpretation selects the HH wavefunction as the correct wavefunction at zeroth order.

• One may think that the issue with the HH wavefunction is intimately related to the unboundedness in the gravity theory. Finding another way to derive tunneling probabilities in Euclidean gravity theory should help to clarify some of the issues. In the stochastic formulation for an inflationary universe developed by Starobinsky and others [16,17], one can derive the tunneling rate from one Λ to another Λ; it is again given by the instanton formalism, not the LV formalism. We shall elaborate on this approach and motivate a correction term (similar to the one due to decoherence effect in path integral approach), shedding further insight to the underlying physics.

• In usual field theory, the tunneling probability is dominated by the most symmetric instanton. Here, it would have been the S^4 instanton. Instead, as argued in Ref [9,10], this is not the case here, leading to the consideration of other complicated instantons, such as wormholes and a chain of S^4 bubbles. This is a consequence of the unboundedness of the Euclidean action of the S^4 instantoin. Naively, the LV wavefunction avoids this problem. However, the LV wavefunction prefers large Λ, in a region where the semi-classical approximation breaks down anyway. In contrast, the proposed modified Euclidean action has a lower bound, and so offers a chance to avoid the disaster encountered by the S^4 instanton. We like to point out that even in quantum theory, the most symmetric instanton will be modified in the presence of an environment. Here, the S^4 instanton would be recovered in the limit where the cut-off (i.e., superstring scale) approaches zero.

Once we adopt the improved HH wavefunction, we find that it can be readily applied to the selection of the original (initial) universe in the cosmic string landscape. The original problem associated with the HH wavefunction (i.e., universes with a zero cosmological constant are preferred) no longer holds. Instead, an inflationary universe seems to be preferred. In particular, a 4-dimensional inflationary universe not unlike ours seems to be a prime candidate with relatively large tunneling probability. If instead we blindly apply the correction to the LV wavefunction, we find that it does not yield anything new, rather, a large cosmological constant universe will again be preferred. In the large cosmological constant case, semi-classical approximation breaks down and this whole approach becomes moot.

It has been argued by DeWitt and others [18] that, to avoid a singularity at $a = 0$, the wavefunction there must vanish, i.e., $\Psi(a = 0) = 0$. This argument becomes more relevant when one considers tunneling from nothing to any stringy vacua, where typically the scale factor $\mathbf{a}$ is a multi-dimensional vector. A non-vanishing Ψ at $\mathbf{a} = \mathbf{0}$ will introduce an arbitrary initial condition, which violates the basic notion of nothing. Any choice of initial condition other than nothing will bias the selection of preferred vacua, and furthermore raise the question of why that particular choice

of initial condition. With $\Psi(a = 0) = 0$, only the HH wavefunction makes sense. To see why this happens, we refer to Fig.(3). In the under-the-barrier region the Wheeler-DeWitt equation yields an exponentially growing and an exponentially decaying mode as its solutions. Clearly, the boundary condition that the wavefunction should vanish at $a = 0$ requires the presence of the growing mode. The growing mode attains a maximum value that is the exponential of the inverse of Λ. One can see why this is so. The base of the under-the-barrier region is proportional to $\sqrt{3/\Lambda}$. Smaller the value of Λ, larger the under-the-barrier region and larger the maximum value that the growing mode attains. This is precisely the HH result. The HH wavefunction goes as $\exp(3\pi/G\Lambda)$. Hence, the condition $\Psi(a = 0) = 0$ is consistent only with the HH wavefunction.

The rest of this note elaborates on some of these points. We shall first give a brief review of the debate, which originated (1) due to the puzzling property of the HH wavefunction applied to the quantum creation of the universe; and (2) from the discrepancy between the instanton approach and the naive WKB approach when applied to this problem. We then elaborate on some of the above arguments on why the instanton method is correct. We discuss the issue in terms of the Wheeler-DeWit (WDW) equation, where the cosmic scale factor a plays the role of the coordinate, and re-examine the physics in the WDW equation.

2. The Debate

In ordinary quantum mechanics one can evaluate the wavefunction of a system by evaluating a path integral with given initial conditions.

$$\psi(x, t) = < x, t | x_i, t_i > = N \int_{x_i, t_i}^{x, t} D[x] e^{iS[x, t]} \tag{2.1}$$

The functional integral is between all possible world lines interpolating between the two boundary conditions. Upon Wick rotation one simply picks up the ground state contribution to the above propagator. In other words, $\psi(x, t)$ becomes the ground state wavefunction. One can do the analogous in quantum gravity and define a ground state wavefunction of the universe. The Hartle-Hawking proposal for the wavefunction of the universe imposes a certain initial condition in the Euclidean spacetime. The initial condition comprises of a null geometry and the final geometry is some three geometry, h_{ij}, and the interpolating geometries are compact four geometries.

$$\Psi(h_{ij}) = \int_{\emptyset}^{h_{ij}} D[g_{\mu,\nu}] e^{-S_E[g_{\mu,\nu}]} \tag{2.2}$$

(The conventions we follow are those of [2]). The Euclidean action is defined as

$$S_E = -\frac{1}{16\pi G} \int d^4 x \sqrt{-g} \, (R - 2\Lambda) \tag{2.3}$$

The Euclidean metric is given by

$$ds^2 = d\tau^2 + a(\tau)^2 d\Omega_3^2 \tag{2.4}$$

With this metric ansatz, the action becomes

$$S_E = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \frac{\Lambda}{3}a^3 \right) \tag{2.5}$$

The Hubble constant squared is given as usual by $H^2 = \Lambda/3$. The dominant contribution to the path integral comes from the saddle point which can be found by looking for the solution of the Euclidean Einstein equation for this minisuperspace model. The result is the S^4 instanton given by

$$a(\tau) = \frac{1}{H} \cos(H\tau) \tag{2.6}$$

This instanton is interpreted as describing the creation of the universe from a state of nothing. Pictorially this is represented in Fig. 1. The lower half shows half the S^4 instanton (in Euclidean spacetime), the upper half shows a deSitter universe (in Minkowski spacetime).

To find the probability of the formation of the deSitter universe by this S^4 instanton one calculates the Euclidean action for the instanton, i.e. $S_E[a(\tau)]$ where $a(\tau)$ is given by Eqn.(2.6). The probability is then simply given by the

$$P_{HH} = e^{3\pi/G\Lambda} \tag{2.7}$$

Assuming that a stringy landscape of vacua exists, one might want to apply this wavefunction as a selection principle to choose a particular deSitter vacuum. Then the above wave function seems to select a vacuum with a vanishing cosmological constant. However, this is precisely the value of Λ where the wavefunction diverges. This is an infrared divergence.

To get around this problem of the Hartle-Hawking proposal, Linde and Vilenkin have proposed alternatives. There are two separate ways to motivate the Linde-Vilenkin wavefunction. In fact, technically (see [12]) the tunneling wavefunction is distinguished from the Linde wavefunction. The reason we put them under the same classification is that they yield the same answer for the probability for the creation of the universe.

To introduce the LV wavefunction, let us consider a closed Robertson-Walker universe filled with a vacuum energy ρ_{vac} . Let a be the scale factor. The evolution equation for a can be written as

$$\pi^2 + a^2 - a^4/a_0^2 = \epsilon \tag{2.8}$$

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where $\pi = -a\dot{a}$ is the momentum conjugate to a and $a_0 = (3/4)\rho_{vac}^{-1/2}$. This equation is identical to that of a particle of energy $E = 0$ moving in a potential $U(a) = a^2 - a^4/a_0^2)$ (see Fig.(2)). In quantum cosmology, there is the possibility of the universe tunneling through the potential barrier to the regime of unbounded expansion. The semiclassical tunneling probability is estimated by Vilenkin using the WKB method

$$P \sim \exp\left(-2\int_{a_1}^{a_2} |\pi(a)|da\right). \tag{2.9}$$

This gives the tunneling probability from nothing to a closed universe of a finite radius a_0. The corresponding probability can be found by setting $a_2 = a_0, a_1 = 0$ in Eq.(2.9) . The result is $P \sim \exp(-3\pi/G\Lambda)$, where Λ and ρ_{vac} are related as usual. The tunneling approach to quantum gravity assumes that the universe originated in a tunneling event of this kind. Once it nucleates, the universe begins a deSitter expansion phase.

The Wheeler-DeWitt equation for this simple model can be obtained by replacing $\pi(a) \rightarrow -id/da$. The resulting equation is

$$\left(\frac{d^2}{da^2} - a^2 + \frac{a^4}{a_0^2}\right)\Psi(a) = 0 \tag{2.10}$$

This equation has outgoing and incoming wave solutions corresponding to expanding and contracting universes in the classically allowed range $a > a_0$, and exponentially growing and decaying solutions in the classically forbidden range $0 < a < a_0$ (Fig.(2)). The boundary condition that selects the tunneling wavefunction requires that Ψ should include only an outgoing wave at $a \rightarrow \infty$. The under-barrier wavefunction is then a linear combination of the growing and decaying solutions. The two solutions have comparable magnitudes near the classical turning point $a = a_0$, but the decaying solution dominates in the rest of the under-barrier region.

In a more realistic model, the constant vacuum density ρ_{vac} is replaced by the potential $V(\phi)$ of some scalar field ϕ. If $V(\phi)$ is sufficiently slowly varying in ϕ, one finds the same result as above, with the replacement $\rho_{vac} \rightarrow V(\phi)$. The highest probability is found for the largest values of $V(\phi)$ (and smallest initial size a_0). Thus, the tunneling wavefunction predicts that the universe is most likely to nucleate with the largest possible vacuum energy.

A similar proposal comes from Linde. Linde suggested that the wavefunction of the universe is given by a Euclidean path integral, with the Euclidean time rotation performed in the opposite sense, $t \rightarrow +i\tau$, yielding

$$\Psi = \int^{(h_{ij},\phi)} e^{+S_E[h_{ij},\phi]} \tag{2.11}$$
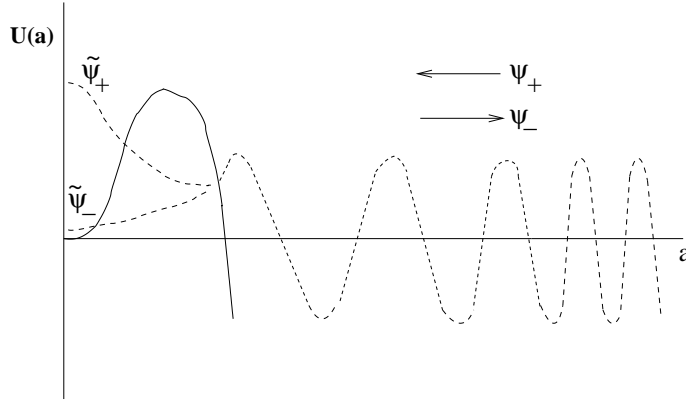
Figure 3: The tunneling wavefunction. The growing and the decaying components under the barrier combine to form the outgoing and incoming waves on the right. If only the growing mode is present (the Hartle-Hawking wavefunction), or if only the decaying mode is present (the Linde wavefunction), there will be both outgoing and incoming waves on the right. Vilenkin demands a particular combination so there is only outgoing wave. Decoherence will yield the outgoing wave as the classical solution.

where h_{ij} is the metric of the 3-geometry that we are interested in and the 3-geometry contains some field ϕ. For the simple metric under consideration, this wave function gives the same nucleation probability as the tunneling wave function.

There is one big problem with this proposal. The path integral, since it is done with the opposite Wick rotation, is divergent over the matter and the inhomogeneous metric modes. This would lead to a catastrophic particle creation after the nucleation of the universe.

Although both the Vilenkin (tunneling) and the Linde proposals give the same answer for the simple model at hand, there is difference in them in terms of the boundary conditions they impose (see [12]). As we have already mentioned, the Wheeler-DeWitt equation (2.10) allows for two independent solutions (the growing and the decaying modes) under the barrier, and two independent solutions (the incoming and the outgoing modes) in the classically allowed region. Vilenkin's proposal includes a linear combination of the growing and the decaying modes under the barrier, and only an outgoing mode in the classically allowed region; whereas the Linde proposal includes only the decaying mode under the barrier and a linear combination of the incoming and the outgoing modes in the classically allowed region. Of course, for both the proposals, the decaying mode is dominant near $a = 0$. This is in keeping with the classical intuition that before tunneling the system (which, in this case, is the universe) should be localized at the origin.

Both the Linde and the Vilenkin proposals favor the nucleation of a universe with the largest value of Λ, which is in some sense is problematic in itself. The semiclassical description breaks down at values of Λ close to the Planck scale. In fact, according to their proposal, quantum foam is more favored over a universe that is closed and inflating.

3. Inclusion of a scalar field

In any realistic model, the universe will contain both gravitational and non-gravitational degrees of freedom. Let us go beyond the minisuperspace (that contains the scale factor $a(\tau)$ only) by including a scalar field ϕ., for example, the inflaton field. Consider Linde's proposal for the wavefunction which involves Wick rotating the time axis in the opposite sense $t \to i\tau$. Such a rotation changes the sign in front of the Euclidean action in Eq.(2.3), and the probability of the creation of the universe is given by $P \sim e^{-3\pi/G\Lambda}$. This proposal has the problem that even though it cures the infrared divergence of the Hartle-Hawking proposal, it makes the scalar field unbounded from below.

To see this, let us consider the scalar field action. In Loretzian signature the action is

$$S[\phi] = \int dt \left(\frac{1}{2}(\partial_\mu \phi)^2 - U(\phi) \right) \tag{3.1}$$

We assume that the scalar field potential is bounded from below. Now, under the usual Wick rotation $t \to -i\tau$, the path integral is given by

$$\int D[\phi] e^{-S_E[\phi]} \tag{3.2}$$

where $S_E[\phi]$ is given by

$$S_E[\phi] = \int d\tau \left(\frac{1}{2}(\partial_\mu \phi)^2 + U(\phi) \right) \tag{3.3}$$

Since, $U(\phi)$ is bounded below and in Euclidean signature the kinetic term is positive definite, the Euclidean action is also bounded from below. However, if one does the opposite Wick rotation $t \to i\tau$, then the action ceases to be bounded from below after the Wick rotation. The path integral is given by

$$\int D[\phi] e^{|S_E[\phi]|} \tag{3.4}$$

Consequently, the scalar field becomes unstable. This was noted in [20]. The scalar field becomes unstable and leads to a catastrophic particle production.

4. The Modified Wavefunction

One of the main motivations for looking for alternatives (i.e., solutions to the WDW equation with different boundary conditions) to the Hartle-Hawking wavefunction as a possible description of the spontaneous creation of the universe was its preference for a universe with a vanishing Λ. In this section we shall argue that once one goes beyond the minisuperspace model, it is natural to expect corrections that will cure this infrared divergence of the wavefunction.

In [21] the authors calculated the metric and scalar field perturbations. The homogeneous degrees of freedom corresponding to the scale factor and the scalar field were treated exactly. The inhomogeneous degrees of freedom were treated to the second order. These inhomogeneities were shown to lead to the temperature anisotropies of the order observed in the CMB. If we sought a description solely in terms of the homogeneous degrees of freedom, then these inhomogeneities were shown to lead to a redefinition of the cosmological constant, the Newton's constant, and the generation of curvature squared counter terms. This is just renormalization of the parameters in the action such that the UV cut-off dependent physics does not show up in the low energy description.

We shall argue in this section that the above is true if one constrains the symmetry of the S^4 instanton (i.e., the deSitter $SO(5)$ rotational symmetry) to remain unbroken upon the inclusion of the inhomogeneous degrees of freedom. However, there is no apriori reason for this. In fact, for a pure deSitter spacetime, where the cosmological constant comes from a scalar field with a flat potential, the large wavelengths of the scalar field fourier modes show up as deSitter fluctuations and couple with the scale factor. We shall show that this backreaction leads to natural metric fluctuations in a deSitter space which lead to a breaking of the $SO(5)$ symmetry and the generation of a radiation term on the inclusion of the inhomogeneous modes (these inhomogeneous modes also lead to a redefinition of the curvature constant K present in the Friedmann equations. But , as usual, K can be reset to $+1$ for the closed universe). In the absence of these metric perturbations the zero point energies of the inhomogeneous modes leads only to the renormalization of the cosmological constant, the Newton's constant, and the curvature squared counterterms. But the metric perturbations, natural in a deSitter spacetime, lead to an additional radiation term. These metric perturbations are similar to the ones discussed in, for example, [16] in the context of stochastic inflation. There the regime where one obtains a Hartle-Hawking like distribution for the scalar field is also the regime where the metric has sizeable fluctuations.

As discussed in Sec. 1, the Euclidean action for the minisuperspace model (for which the metric is given by Eq.(2.4)) is given in terms of the scale factor by Eq.(2.5).

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This, in turn, leads to the Hartle-Hawking result Eq.(5.1). Let us include metric perturbations. The perturbed three metric is given by

$$h_{ij} = a^2 \left(\Omega_{ij} + \epsilon_{ij} \right) \tag{4.1}$$

where Ω_{ij} is the metric on the unit three-sphere and ϵ_{ij} is a perturbation on this metric and can be expanded in spherical harmonics over S^3 [21]. For simplicity let us consider only the tensor perturbations. The tensor perturbations, with amplitudes t_n, have the Euclidean action

$$S_E^n = \frac{1}{2} \int t_n \hat{D} t_n + \text{ boundary term} \tag{4.2}$$

where

$$\hat{D} = \left(-\frac{d}{d\tau} \left[a^3 \frac{d}{d\tau} \right] + a(n^2 - 1) \right) \tag{4.3}$$

where the background satisfied the classical equation of motion. The action is extremized when t_n satisfies the equation $\hat{D} t_n = 0$, i.e. the equation

$$\frac{d}{dt} \left[a^3 \dot{t}_n + (n^2 - 1)a t_n \right] = 0 \tag{4.4}$$

For t_n that satisfies the equation of motion, the action is just the boundary term

$$S_E^{n(cl)} = \frac{1}{2} a^3 \left(t_n \dot{t}_n + 4 \frac{\dot{a}}{a} t_n^2 \right) \tag{4.5}$$

The path integral over t_n will be

$$z_n = \int d[t_n] \exp(-S_E^{n(cl)}) = (\det \hat{D})^{-1/2} \exp(-S_E^{n(cl)}) \tag{4.6}$$

Now let us perform the path integral over the superspace comprising the scale factor and the tensor modes. We shall get the following path integral:

$$Z = \int D[a] e^{-S_E} \prod_n z_n \tag{4.7}$$

where S_E is the Euclidean action for the scale factor given by Eq.(2.5). If we are interested only in the dynamics of the scale factor and not the high frequency tensor modes, then we simply trace them out. Notice that the tensor mode classical action $S_E^{n(cl)}$ depends on boundary conditions. Let us label the initial and the final value of the tensor perturbations as t_n^i and t_n^f, respectively. To take the trace we set $t_n^i = t_n^f$, and integrate over all possible values of t_n^i. This is nothing but an integral over the propagator with periodic boundary conditions and it gives the sum over the various

eigenvalues of the simple harmonic oscillator corresponding to the tensor mode (of course, with a variable mass and frequency) [23, 24].

$$\int dt_n^i \int dt_n^f \delta(t_n^i - t_n^f) z_n = \frac{1}{e^{D_n/2} - e^{-D_n/2}} \tag{4.8}$$

where D_n is the integrated frequency and $D_n/2$ is the ground state energy of the oscillator,

$$D_n = \int d\tau \frac{\sqrt{n^2 - 1}}{a(\tau)} \simeq \int d\tau \frac{n}{a(\tau)} \tag{4.9}$$

The denominator $a(\tau)$ in the frequency is simply the redshift factor. So tracing out the tensor perturbations, one gets the path integral

$$\int D[a]e^{-S_E} \prod_n \frac{1}{e^{D_n/2} - e^{-D_n/2}} \simeq \int D[a]e^{-S_E} \prod_n \left(e^{-D_n/2} + e^{-3D_n/2} + ... \right)$$

$$\simeq \int D[a]e^{-S_E - \sum_n D_n/2} = \int D[a]e^{-S_E^{mod}} \tag{4.10}$$

where only the dominant term is kept.

Let us now look at the counting of modes. From Eq.(4.9) and Eq.(4.10), it is clear that the modified action is given by

$$S_E^{mod} = S_E + \sum_n D_n/2 = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \frac{\Lambda}{3}a^3 + \frac{1}{a(\tau)} \sum_n n \right) \tag{4.11}$$

The index n really stands for n, l, m that represent the spherical harmonics on S^3. As explained in [4], this leads to the following result

$$\sum_n n \simeq \int_0^N dn n \times n^2 = \frac{N^4}{4} \tag{4.12}$$

where the factor of n^2 shows the degeneracy arising due to the indices l and m for a given n. N is given by

$$N = \frac{H^{-1}}{l_s} f(a) \tag{4.13}$$

that is, we include the wavelengths that lie between string scale (UV cut-off), and the Hubble scale (IR cut-off). The function $f(a)$ is a fourth degree polynomial in $a(\tau)$ that we shall discuss now.

- Maintaining the $O(5)$ symmetry :
  The simplest counting is given by $f(a) = a$ in Eq.(4.13). The radius of S^3

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grows from zero (at the south pole of S^4) to a maximum value at the equator, to back to zero at the north pole. So when one counts the modes, one might simply take $N = \frac{H^{-1}}{l_s} a(\tau)$.

From Eq.(4.11) one can see that this leads to a renormalization of the cosmological constant from the ground state energies of the tensor modes:

$$\Lambda \to \Lambda + \frac{N^4}{4} \qquad (4.14)$$

where $\frac{N^4}{4} \equiv \frac{1}{4(Hl_s)^4}$. Of course, a proper calculation using renormalization techniques will also give $\log(H/M_P)$ terms. This is well known.

• Breaking the $O(5)$ symmetry:
However there is no reason to restrict oneself to maintaining the $O(5)$ symmetry. In fact, as explained in [21], to mainitain the $O(5)$ symmetry, the perturbations must vanish at the poles of the S^4 geometry. Since we are considering only the tensor modes, $t_n = 0$ at the poles. This is very much possible at the classical level. However, once we treat the perturbations quantum mechanically, then even assuming that the perturbations start off in their ground states at the south pole of S^4 (as in [21]) it is easily seen that the amplitudes of the perturbations do not vanish at the poles. The amplitude of the oscillators has a nonzero value in their ground states. In fact, as one approaches the poles, the oscillator conributions backreact the geometry more and more till the WKB approximation used in [21] breaks down. One expects some kind of flattening of the poles due to this backreaction. This will lead to the breaking of the $O(5)$ symmetry.

If we allow for this breaking of the symmetry, then $f(a)$ will assume a general form:

$$N^4 \propto f(a)^4 = Aa^4 + Ba^3 + Ca^2 + Da + E \qquad (4.15)$$

One can see from Eqs. (4.11, 4.12) that the E-term will lead to a radiation term in the modified action. The Da term will lead to a matter term . Such a matter term was anticipated in [3] and leads to a $\frac{1}{\Lambda^{3/2}}$ correction to the Hartle-Hawking result. The Ca^2 term leads to a redefinition of the curvature. Recollect that if we retain a curvature K term from the Friedmann equation, then the a term in Eq.(4.11) should really be written as Ka. That is, the unmodified Lagrangian is $\frac{1}{2}(-a\dot{a}^2 - Ka + \frac{\Lambda}{3}a^3)$. The Ca^2 term just leads to a redefinition of K, which can be reset to $K = +1$ for closed universe such as the one we are studying. The Aa^4 term, as we saw in the previous discussion, simply renormalizes the cosmological constant. From the previous discussion we expect $A = 1$. As we

shall explain now, we expect the B term to be equal to zero.

To get an estimate of these coefficients, let us consider a more familiar situation in an inflationary universe where the cosmological constant is supplied by a scalar field potential $V(\phi)$. During the deSitter phase, the kinetic term is negligible. However, we know that in a deSitter spacetime there are fluctuations of scalar field given by $\delta\phi = H/2\pi$. Unless the inflaton field is slowly rolling close to the end of the inflation, the inflaton will typically undergo fluctuations of the order H in a time period H^{-1} [17]. So $\dot{\phi} \sim H^2$. The corresponding metric fluctuation is given by $\delta a/a = H\delta\phi/\dot{\phi} \simeq 1$. This can be seen in terms of metric perturbations. Let us choose a gauge in which the metric perturbations occur only in the g_{ij} - i.e. the spatial components. So the unperturbed metric goes to a perturbed one

$$a^2 \delta_{ij} dx^i dx^j \rightarrow a^2[(1+2D)\delta_{ij} + 2E_{ij}]dx^i dx^j \tag{4.16}$$

where we are following the notations and treatment of [15]. One can separate the above perturbations into scalar and tensor parts as follows,

$$
\begin{aligned}
a^2[(1 &+ 2D)\delta_{ij} + 2E_{ij}]dx^i dx^j \\
&= a^2 \delta_{ij} dx^i dx^j + 2Da^2 \delta_{ij} dx^i dx^j + 2a^2 E_{ij} dx^i dx^j \\
&= a^2 dx^i dx_i + 2Da^2 \delta_{ij} dx^i dx^j + 2a^2 \left(-\frac{k_i k_j}{k^2} + \frac{1}{3}\delta_{ij}\right) E dx^i dx^j \\
&= a^2 dx_i dx^i + 2a^2 \left(D + \frac{1}{3}E\right)\delta_{ij}dx^i dx^j - 2Ea^2 \frac{k_i k_j}{k^2} dx^i dx^j
\end{aligned}
\tag{4.17}
$$

The scalar metric perturbation R is given by

$$R = D + \frac{1}{3}E \tag{4.18}$$

Considering only the scalar perturbation to the scale factor, we can write

$$R = \delta(a^2)/2a^2 = \delta a/a \tag{4.19}$$

Furthermore, the term $D + \frac{1}{3}E$ is related to the scalar field perturbation by

$$D + \frac{1}{3}E = H\frac{\delta\phi}{\dot{\phi}} \tag{4.20}$$

Hence

$$\delta a/a = H\delta\phi/\dot{\phi} \simeq 1 \tag{4.21}$$

So, $\delta a \simeq H^{-1}$ over one Hubble time. To get an estimate of the coefficient E, let us write $f(a)^4$ as

$$f(a)^4 = \; <(a(\tau) + \delta a)^4> \tag{4.22}$$

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where $< ... >$ is the expectation value. Since the de Sitter fluctuations are random, we have $< \delta a >=< \delta a^3 >= 0$. So we simply have a radiation term apart from the renormalization of Λ and the curvature term K. Here, $E \sim 1$.

The metric perturbation may be studied in the same way. This tensor mode fluctuation has amplitude v_k. In an inflationary universe, such modes leave the horizon as $k = aH$, which is then frozen until they re-enter the horizon. They may be detected via their effect on the polarization of the cosmic microwave background radiation. Since the tensor mode is responsible for the primordial B mode polarization, detecting the B mode is the main goal of future CMB observations. In our calculation, we are simply integrating over this tensor mode fluctuations. These tensor modes are massless, so they contribute to the radiation in the deSitter space, thus breaking the deSitter symmetry. This is spontaneous symmetry breaking by radiative corrections. In Euclidean metric, they modify the S^4 instanton in a way such that the $SO(5)$ rotational symmetry is broken. Since the universe emerging from the tunneling is a closed universe, there are no modes with wavelength longer than the size of the universe, i.e., $\Lambda^{-1/2}$. So there are no superhorizon modes. The divergence at $k \rightarrow \infty$ requires the introduction of a cut-off M_s, which is naturally provided by the string scale in string theory. So the sum over modes yields a term $\sim M_s^4/\Lambda^2$.

Since a pure quantum deSitter space does not exist, we cannot consider its tunneling from nothing. Instead, consistency requires us to consider the tunneling to a deSitter space with a radiation term. This radiation term receives contributions from the metric perturbation as well as from the quantum fluctuations of all matter fields, so it depends on the specific vacuum the tunneling process is heading. This leads to the modified action

$$S_E^{mod} \equiv -F = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \frac{\Lambda}{3}a^3 + \frac{9bM_s^4}{4a(\tau)\Lambda^2} \right) \qquad (4.23)$$

where M_s is the string mass scale, and Λ is now the redefined cosmological constant. Here b measures the degrees of freedom in the approximate deSitter space. The curvature K has been set to $+1$. We have defined F here as a "free energy". The inclusion of the radiation term generalizes the entropy to free energy (free energy per unit temperature, to be precise). The equation of motion is now given by the variation of the modified action (setting $b = 1$ for the moment) [4]

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \Lambda - \frac{9M_s^4}{4\Lambda^2 a^4} \qquad (4.24)$$

the solution to which is given by

$$a(\tau) = \frac{1}{\sqrt{2}H} \sqrt{\left(1 + \sqrt{\left(1 - \frac{2GM_s^4}{3\pi H^2} \right) \cos(2H\tau)} \right)} \qquad (4.25)$$
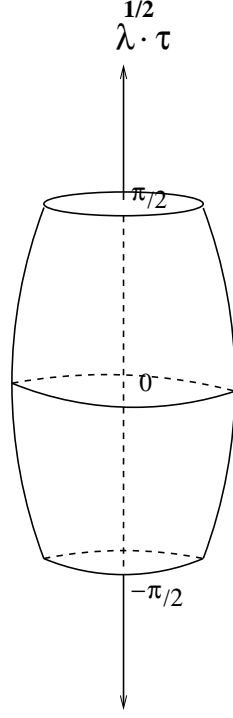
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Figure 4: The Modified Bounce : It is a squashed version of S^4 resulting in a "barrel". Topologically, it is still a S^4. Presumably, higher order corrections will smooth the edges of the barrel, probably resulting in an ellipsoid.

This is an interesting geometry. Although it has radiation, it does not encounter any singularity.

The modified result is then given by

$$P \simeq e^{-F} = \exp\left(\frac{3\pi}{G\Lambda} - \frac{6M_s^4}{\Lambda^2}\right) \tag{4.26}$$

The probability of the nucleation of the universe from nothing is now determined by *the minimization of a free energy.*

5. A Problem of the Hartle-Hawking Wavefunction and Its Possible Resolution

Recall that the HH wavefunction for deSitter space is given by

$$\Psi_{HH} \simeq \exp(3\pi/2G\Lambda) \tag{5.1}$$

For dynamical Λ, this implies that $\Lambda \to 0$ is much preferred. Since the cosmic scale factor a which measures the size of the universe, behaves as $a \simeq 1/\sqrt{\Lambda}$, this implies

the creation of an infinite size (Minkowski) universe is exponentially preferred. Based on our knowledge of quantum mechanics, a macroscopic object behaves classically, so tunneling, a quantum process, should be very much suppressed. So this poses a puzzle. Phenomenologically, our universe has gone through the big bang epoch and most probably an inflationary phase before that, so the prediction of the HH wavefunction is inconsistent with observation.

The above intuitive argument has been laid out in many papers [9]. Clearly, there is a serious problem associated with the HH wavefunction. Here, we like to point out that this problem associated with the HH wavefunction may disappear when it is replaced by the improved wavefunction. Let us first review the problem and then show how it may be resolved.

To quantify this problem more precisely, let us consider the argument in Ref. [10]. If tunneling via a S^4 instanton has such a large probability, then tunneling via a chain of connected S^4s is even more likely, with wavefunction

$$\Psi \sim \exp\left(k\frac{3\pi}{2G\Lambda\hbar} - [k-1]\frac{\Delta}{G\hbar}\right) \tag{5.2}$$

where Δ is the contribution from the small neck connecting the S^4s, $\Delta \sim r^2$, with r the size of the neck, and k is the number of S^4s in the chain. Any reasonable estimate of Δ yields a finite value that depends only weakly on Λ. Although Δ leads to some suppression, the first term in the exponent dominates over the Δ term as $\Lambda \to 0$. As a result, the tunneling probability $|\Psi|^2$ is dominated by the large k contributions. This leads to a pathology as $k \to \infty$.

Even if k has a maximum (one may argue that $k_{max} \sim \exp(3\pi/G\Lambda)$), the HH wavefunction has a serious problem. Consider a point p in one of the deSitter spaces and compare the amplitude for the geomtery at p to be smooth versus that to find a Schwarzschild wormhole end (i.e., a black hole) at p. The leading term overwhelms the suppression so the result predicts that black holes are dense in Minkowski space-time. As pointed out in Ref. [10], this is the best case scenario. It could be that the wavefunction is too divergent to make any sense.

The above disaster probably happens to any quantum theory where the S_E is unbounded from below. It is usually assumed that the most symmetric instanton solution yields the lowest Euclidean action. This is clearly violated here. This is related to the unboundedness from below of the S^4 Euclidean action.

Now consider the improved wavefunction. Here, we now have

$$\Psi \simeq \exp\left(-kF - [k-1]\frac{\Delta}{G\hbar}\right) \tag{5.3}$$

Now, if $F + \Delta/G > 0$, or

$$\Delta > -GF = \frac{3\pi}{2\Lambda} - \frac{6M_s^4 G\hbar}{2\Lambda^2} \tag{5.4}$$

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then the single barrel term, that is, $k = 1$ term dominates. If Δ is independent of Λ, then the right hand side of the above inequality can be replaced by its value at Λ_{max}, the value that maximizes the tunneling rate for a single barrel, i.e., $\Delta > 3\pi/4\Lambda_{max}$. If Δ depends on Λ, the above inequality must be satisfied for all allowed Λ, not just Λ_{max}. That is, if the inequality (5.4) is not satisfied for some allowed Λ, then the tunneling probability will increase as k increases, so for $k \to \infty$, the tunneling rate blows up.

It is difficult to estimate Δ reliably. To get an idea of the dimensionality of Δ here, suppose $r \simeq a$ at the end of the barrel. This yields $\Delta \sim a^2 \sim 1/(GM_s^2)^2$. Since F scales in the same way in general, $F \simeq 1/(GM_s^2)^2$, a more careful analysis is required to see if $F + \Delta/G > 0$ is satisfied or not. So we see that the improved wavefunction may not suffer from the problem that plagues the HH wavefunction. Clearly, a more careful examination of this issue will be very important.

In usual field theory, the tunneling probability is dominated by the most symmetric instanton. If this is the case here, it would have been the S^4 instanton. Instead, as argued in Ref [9], this is not the case here. This leads the consideration of wormholes and the above chain of bubbles. This is a consequence of the unboundedness of the Euclidean action of the S^4 instantoin. In contrast, the barrel action has a lower bound, and so offers a chance to avoid the disaster encountered by the S^4 instanton. In contrast, the single barrel is in some sense more symmetric than the chain of bubbles. This offers the hope that the single barrel dominates the tunneling probability.

6. Hartle-Hawking Distribution from Stochastic Inflation

We shall give a quick review of stochastic inflation. The reader should refer to [16, 17, 19] for details. The basic idea is that the dynamics of a large-scale quasi-homogeneous scalar field producing the de Sitter stage is strongly affected by small-scale quantum fluctuations of the same scalar field and, hence, becomes stochastic. The evolution of the corresponding large-scale space-time metric follows that of the scalar field and is stochastic too. One can write down a Langevin equation for the long wavelength modes of the scalar field and a corresponding Fokker-Planck equation for the distribution function for the large wavelength modes.

Let the de Sitter phase be produced by the potential of some scalar field with the Lagrangian density

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{6.1}$$

The Hubble constant is given by

$$H^2 = \frac{8\pi G V(\phi)}{3} \tag{6.2}$$

At the de Sitter phase $H \simeq H_0 = $ constant, and the scale factor grows as $a(t) = a_0 \exp(H_0 t)$. It is natural during the de Sitter phase to separate the full scalar field ϕ into a long wavelength part $\bar{\phi}$ and the short wavelength part

$$\phi = \bar{\phi}(t, \vec{r}) + \int \frac{d^3 k}{(2\pi)^{3/2}} \, \theta(k - a(t)H_0) \, [\hat{a}_k \varphi_k(t) e^{-i\vec{k}\cdot\vec{r}} + \hat{a}_k^\dagger \varphi_k^*(t) e^{i\vec{k}\cdot\vec{r}}] \qquad (6.3)$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the usual creation and annihilation operators. Here $\bar{\phi}(t, \vec{r})$ contains only the long wavelength modes with $k < H_0 a(t)$. The second integral term satisfies the free massless scalar wave equation in the deSitter background : $\Box\varphi = 0$ (assuming that the mass of the scalar field is much smaller than H). The solution is well known

$$\varphi_k = H_0(2k)^{-1/2} \left(\eta - \frac{i}{k}\right) \exp(-ik\eta); \eta = \int \frac{dt}{a(t)} = -\frac{1}{a(t)H_0} \qquad (6.4)$$

The scalar field ϕ satisfies the operator equation of motion $\Box\phi + \frac{dV}{d\phi} = 0$ exactly. Using Eq.(6.3,6.4) and the slow roll conditions, one obtains the following equation of motion for $\bar{\phi}$ in the leading order

$$\dot{\bar{\phi}}(t, \vec{r}) = -\frac{1}{3H_0} \frac{dV(\bar{\phi})}{d\bar{\phi}} + f(t, \vec{r}) \qquad (6.5)$$

where $f(t, \vec{r})$ is a "noise" term given by

$$f(t, \vec{r}) = -i\frac{a(t)H_0^2}{(2\pi)^{3/2}} \int d^3 k \, \delta(k - aH_0) \, \frac{H_0}{\sqrt{2}k^{3/2}} [\hat{a}_k \exp(-i\vec{k}.\vec{r}) - \hat{a}_k^\dagger \exp(i\vec{k}.\vec{r})] \quad (6.6)$$

The large scale scalar field $\bar{\phi}$, therefore, changes not only due to the classical force $dV(\bar{\phi})/d\bar{\phi}$ but also due to the flow of initially small-scale quantum fluctuations across the deSitter horizon $k = a(t)H_0$ in the process of expansion. This, then, is a stochastic process. Eq.(6.5) is the Langevin equation governing the evolution of $\bar{\phi}$. The evolution of inhomogeneous modes is linear inside the deSitter horizon ; on the other hand, the evolution of $\bar{\phi}$ is non-linear. Note that there are no spatial derivatives in Eq.(6.5) at all. This means that if we are just interested in the evolution of $\bar{\phi}$, and not in what goes on inside the Hubble patch, then we can consider the Hubble patch as a point. The temporal evolution of $\bar{\phi}$ is slow compared to H^{-1} so only processes with characteristic time H^{-1} need to be considered.

Even though $\bar{\phi}$ and $f(t, \vec{r})$ have a complicated operator structure, all terms commute with each other, so we can consider $\bar{\phi}$ and $f(t, \vec{r})$ as classical. But they are stochastic because one cannot assign any definite numerical value to the combination $[\hat{a}_k \exp(-i\vec{k}.\vec{r}) - \hat{a}_k^\dagger \exp(i\vec{k}.\vec{r})]$. The two point function for the noise term can be calculated

$$< f(t_1, \vec{r}) f(t_2, \vec{r}) >= \frac{H_0^3}{4\pi^2} \delta(t_1 - t_2) \qquad (6.7)$$

Thus $f(t)$ has the properties of white noise.

One can also derive the Fokker-Planck equation corresponding to the Langevin equation for $\bar{\phi}$. One is interested in the average value $< F(\bar{\phi}) >$ of some arbitrary function F. One introduces a normalized probability distribution $\rho(\bar{\phi}, t)$, $\int_{-\infty}^{\infty} \rho(\bar{\phi}, t) d\bar{\phi} = 1$, such that

$$< F(\bar{\phi}) >= \int_{-\infty}^{\infty} \rho(\bar{\phi}, t) F(\bar{\phi}) d\bar{\phi} \tag{6.8}$$

The Fokker-Planck equation for ρ is given by

$$\frac{d\rho}{dt} = \frac{H_0^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \bar{\phi}^2} + \frac{1}{3H_0} \frac{\partial}{\partial \bar{\phi}} \left(\frac{dV}{d\bar{\phi}} \rho \right) \tag{6.9}$$

One has to specify the initial condition ρ_0 at some initial time t_0 in order to solve this equation. One should note that this Fokker-Planck equation is applicable only during the slow-roll stage. Later the second time derivative will become important.

6.1 The Hartle-Hawking Distribution from Stochastic Inflation

One has to specify the initial condition ρ_0 in order to be able to solve the Fokker-Planck equation for the distribution function $\rho(\bar{\phi}, t)$. So how can one find something like the Hartle- Hawking wavefunction which corresponds to "no boundary"? A natural idea is to look for stationary solutions that are independent of t. It is easy to solve Eq.(6.9) with $d\rho/dt = 0$. One gets the following solution

$$\rho = \rho_0 V(\phi)^{-1} e^{3\pi/G\Lambda} \tag{6.10}$$

where $\Lambda = 8\pi G V(\phi)$ is the cosmological constant. This is precisely the square of the Hartle-Hawking wavefunction.

It is very remarkable that the stationary distribution coincides with the distribution from Hartle-Hawking wavefunction. Eventually the scalar field will start rolling in some Hubble patches and inflation will end in those patches. However, one should note that the stationary distribution corresponds to that phase when the noise term $f(t, \vec{r})$ is of the same strength as the classical term $dV/d\bar{\phi}$. This leads to the important conclusion that the phase to which the HH distribution is applicable is the one when the stochastic term is non-negligible. During this phase the fluctuations in the metric is of the order of the Hubble parameter $\delta a \sim H^{-1}$.

6.2 Metric Fluctuations during stochastic inflation and mode counting

The fact that the regime of stochastic inflation where the Hartle- Hawking distribution is a solution of the Fokker-Planck equation also has big metric fluctuations will

have important consequence on the mode counting. As was already noted in [16], the space-time metric during stochastic inflation can be written as

$$ds^2 = -dt^2 + e^{h(\vec{r})}a^2(t)(h_{ij}dx^i dx^j) \tag{6.11}$$

where $h(\vec{r})$ is not small and $a(t)$ is the scale factor for a strictly isotropic and homogeneous solution. The quantity $h(\vec{r})$ is essentially stochastic, its r.m.s. value being of the order of its average. If the conditions of the slow-roll inflation are satisfied, some Hubble patch eventually exits the stochastic inflation phase and rolls down to end the inflation (this just the idea of eternal inflation). During this last "useful" part of inflation, sufficiently large regions are produced with the degree of perturbations that matches the observations. During this part, the stochastic noise term in Eq.(6.5) is negligible and the evolution is governed by the classical force term $dV/d\bar{\phi}$. This is just the slow-roll inflationary stage. The inhomogeneities produced at this stage are small and are given by

$$h(\vec{r}) = -2H_0 \delta\bar{\phi}/\dot{\bar{\phi}} \tag{6.12}$$

which gives the usual result $\delta a^2/a^2 = -2H_0\delta\bar{\phi}/\dot{\bar{\phi}}$.

Hence, during the phase of stochastic inflation when the distribution is given by the Hartle-Hawking result, the metric perturbations are given by $\delta a/a \sim 1$. Whereas, it is only much later, during the slow rolling phase, that the much smaller density perturbations corresponding to observations are produced.

This has important consequence for the mode counting. The number of modes between the string scale and the Hubble scale were given by

$$N^4 = \left(\frac{H^{-1}a(t)}{l_s}\right)^4 \tag{6.13}$$

This just led to the redefinition of the cosmological constant. However, taking into account the metric fluctuations, the mode counting is now done as

$$N^4 = \left(\frac{H^{-4} < (a(t) + \delta a)^4 >}{l_s^4}\right) \tag{6.14}$$

If the fluctuations are random, then $< \delta a > = < \delta a^3 > = 0$. This is the proper prescription for mode counting for a purely stochastic fluctuation as this. The result is a radiation term. This radiation term backreacts and modifies the S^4 instanton and gives a barrel solution.

7. The Wheeler-DeWitt Equation and the Hartle-Hawking Wavefunction

The Wheeler-DeWitt (WDW) equation is the Schrodinger like equation for the universe. Just like the Schrodinger equation is a way to impose the Hamiltonian energy

condition at the quantum level, the WDW equation is a prescription to impose the Hamiltonian constraint at the quantum gravitational level.

The Wheeler-DeWitt (WDW) equation implements the Hamiltonian constraint at the quantum level. It is given by

$$H\Psi = 0 \tag{7.1}$$

where H is the total Hamiltonian of the universe.

The Hartle-Hawking wavefunction is a solution of the WDW equation with the following boundary condition at $a = 0$

$$\Psi_{HH}(a = 0) = 0 \tag{7.2}$$

(See Fig.(2)). In the classically allowed region Hartle-Hawking proposal demands the presence of both the incoming wave and the outgoing wave.

The boundary condition Eq.(7.2) has a few virtues to begin with. First, the singularity at $a = 0$ is avoided by this boundary condition [2, 18]. Whereas, in the Linde and Vilenkin cases, the wavefunction has a finite value at $a = 0$, and it is hard to see how the universe would be metastable at $a = 0$. In fact, when one considers the quantum tunneling of a system from a metastable vacuum to a true vacuum in usual quantum field theory, both the metastable vacuum and the true vacuum should be obtainable as solutions of the classical equation of motion. In such a case it would seem natural to impose a Linde/Vilenkin like boundary condition at $a = 0$. As we prepare the system in a metastable vacuum state and leave it there to sit classically (hence, the wavefunction would be nonvanishing at the metastable vacuum), we would like to ask for the probability of quantum tunneling. However, for a deSitter universe (described by $a(t) = H^{-1}\cosh(Ht)$), $a = 0$ is not a classical solution for any value of $\Lambda = 3H^2$ and time. So it is hard to see the meaning of a boundary condition where the wavefunction $\Psi(a = 0)$ does not vanish. Furthermore, a non-vanishing $\Psi(a = 0)$ disagrees with our notion of a unique "nothing". The Hartle-Hawking proposal (Eq.(7.2)) is more natural in this situation. With $\Psi_{HH}(a = 0) = 0$, we have a unique "nothing".

The landscape is a vast collection of vacua with different properties (like the cosmological constant, various gauge field couplings, etc) [22, 25, 26]. In the context of the application of the wavefunction of the universe as a selection criterion on the stringy landscape, Eq.(7.2) is probably the only boundary condition that makes sense. In such a case, Eq.(7.2) allows for a democratic comparison of various vacua. The no boundary proposal obviates the necessity of any initial conditions [2]. One does not have to pick an initial vacuum in the landscape and then wonder why the universe ended up in that particular vacuum.

8. Some Comments

There has been some recent works in applying quantum cosmology to the cosmic landscape [27–32, 34, 35]. In [28] the authors motivate the modified Hartle-Hawking wavefunction using string thermodynamic arguments and the applicability of effective field theory description. It is interesting to note that their result derived from rather general arguments. It is possible that a stringy thermodynamic phase in the very early universe might lead to a radiation like the component that we have seen will modify the wavefunction. Ref [31] discusses possible observational effects of the modified wavefunction and also that chaotic eternal inflation might be highly constrained by the use of the modified wavefunction. It is natural to believe that the usual initial state of a scalar field in the Bunch-Davies vacuum will recieve corrections due to the modified wavefunction. This will be akin to the transplanckian physics discussed by various authors (see [36–38] and references therein). The barrel instanton background is simply deSitter plus radiation in Lorentzian signature metric. Doing a quantum field theoretic calculation for the scalar field in this background will shed light on possible changes in CMB spectrum. This is work is progress [8]. It is, in fact, very interesting to note that a source term arises very naturally when a physical (not comoving) momentum cut-off is imposed on a de Sitter background [36, 39, 40]. The radiation term that leads to the modified wavefunction and that depends on the cut-off might be seen as such a source term. This possibility deserves further investigation.

It is natural question to ask what sense there is to calculating probabilities when one is comparing different universes. Everett and Wheeler [33] were led to their formulation of this problem and a possible solution while asking these questions. If the wavefunction of the universe is to be used to calculate probabilities that involve different universes on a stringy landscape, then one is naturally confronted by these issues. Further work is required along the directions set by [33].

Finally, we would like to address the criticism of the modification of the wavefunction mentioned in [34]. The authors raised the possibility of interpreting the radiation term simply as a renormalization of the cosmological constant due to the vacuum energies of the perturbative modes. This, however, is not true. First, a string scale appears directly in the correction term, thus introducing a new scale into the tunneling formula. As we have discussed in Sec.(4.1), the perturbative modes not only renormalize the constant, but also introduce a radiation term due to the quantum fluctuations of the metric inherent in a de Sitter spacetime. The fluctuations of the background lead to the radiation term that cannot be absorbed into any counterterm. QFT in curved background assumes a fixed background and the matter field is a higher order correction to it. This enables one to do a semiclassical treatment and

absorb away the cut-off dependent quantities. However, having claimed that when the background metric itself is fluctuating, as in a de Sitter spacetime, there will be a source term.

Acknowledgments

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